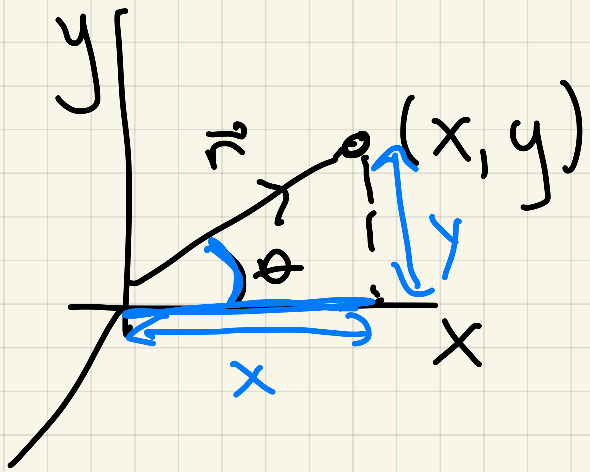
Circular motion

ex: Object spun in a circle on the end of a string.

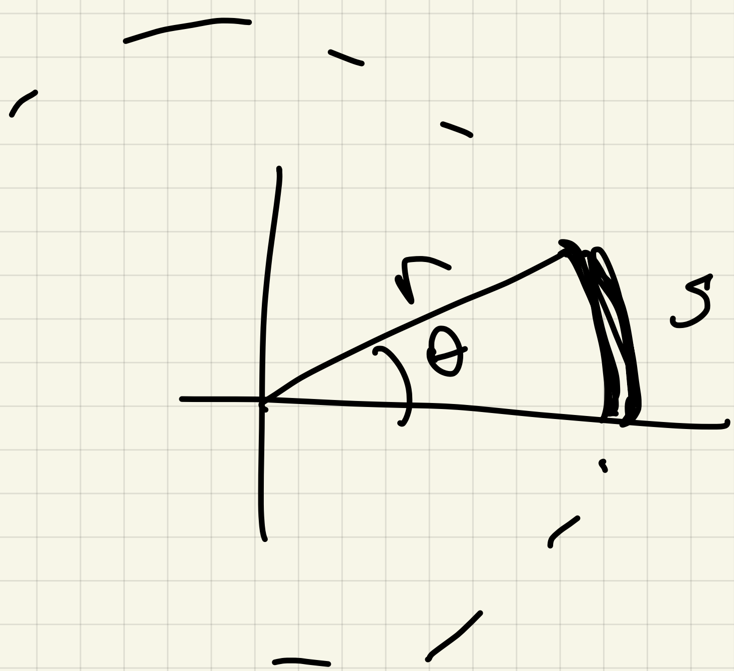


position vector

$$\vec{r} = \begin{bmatrix} x \\ y \end{bmatrix}$$

$$x = r\cos\theta$$

$$y = r\sin\theta$$



s = arclength

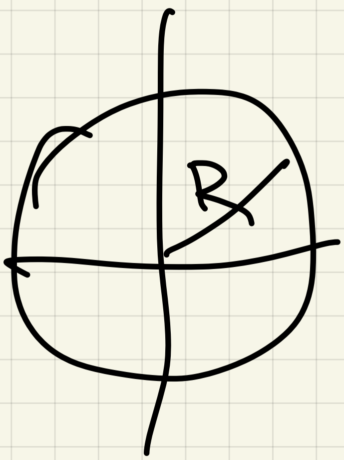
$$s = r\theta$$

circumference

$$C = r(2\pi)$$

For circular motion, s (arclength) is the path of the object.

$$v = \text{derivative of position}$$



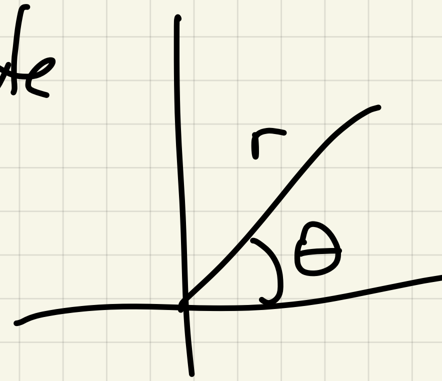
$$v = \frac{ds}{dt} = \frac{d}{dt}(r\theta)$$

For circles, $r = R$
a constant

$$v = R\frac{d\theta}{dt}$$

Greek omega $\boxed{\omega \equiv \frac{d\theta}{dt}}$ θ = angle of polar coordinate

θ ≡ polar coordinate



≡ angular position

For a circle where $r = R$ a constant, θ defines position

ω ≡ angular velocity

is a vector

for now, CW or CCW

units $\frac{\text{rad}}{s}$ ← radians are a unitless unit

$$S = R\theta$$

$\downarrow$ m $\quad \downarrow$ m

Def $1 \text{ rad} = \dfrac{\text{arc length } s}{R}$

$\pi \text{ rad} = 180°$

Uniform Circular motion

$\hookrightarrow r = R = \text{constant}$

ω = is a constant

We can write $\boxed{\theta = \omega t}$

only for uniform circ. motion

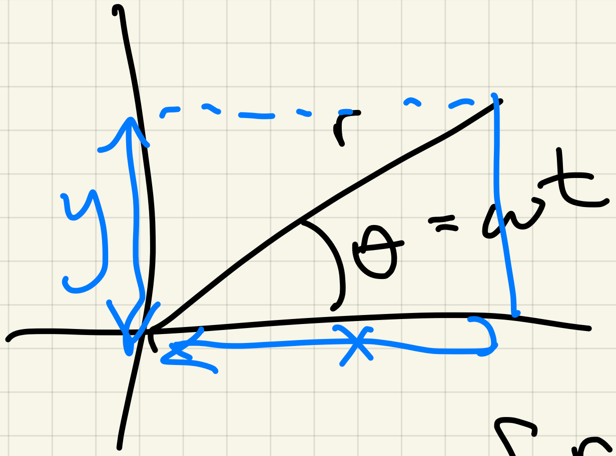
Preview Ch 8

$$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$$

ang velocity

angular acc.

~~Ch 8~~



$$\vec{r} = \begin{bmatrix} r\cos\theta \\ r\sin\theta \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix}$$

uniform circular

$r = R$ a const

$\theta = \omega t$ where $\omega = \text{const}$

$$\vec{r} = \begin{bmatrix} R\cos(\omega t) \\ R\sin(\omega t) \end{bmatrix}$$

velocity is derivative

$$\vec{v} = \frac{d\vec{r}}{dt} = \begin{bmatrix} -\omega R\sin(\omega t) \\ +\omega R\cos(\omega t) \end{bmatrix}$$

Q: Magnitude of velocity?

$$|v|^2 = v_x^2 + v_y^2$$

$$= \left[-\omega R \sin(\omega t)\right]^2 + \left[\omega R \cos\omega t\right]^2$$

$$= \omega^2 R^2 \sin^2(\omega t) + \omega^2 R^2 \cos^2 \omega t$$

$$= \omega^2 R^2 \underbrace{\left[\sin^2(\omega t) + \cos^2(\omega t)\right]}_{1}$$

$$v^2 = \omega^2 R^2$$

$$\boxed{v = R\omega} \Leftarrow \text{magnitude only}$$

same answer from s (arclength)

Back to velocity vector to get acc vector.

$$\vec{v} = \begin{bmatrix} -\omega R \sin(\omega t) \\ \omega R \cos(\omega t) \end{bmatrix}$$

$$\vec{a} \equiv \frac{d\vec{v}}{dt} = \begin{bmatrix} -\omega^2 R \cos(\omega t) \\ -\omega^2 R \sin(\omega t) \end{bmatrix}$$

$$\boxed{\vec{a} = -\omega^2 \vec{r}}$$

acceleration

ω^2 const

original position vector $\Rightarrow$ along a radius of circle

$\vec{r}$ = points radially outward

$-\vec{r}$ = toward center of circle

$\Rightarrow$ <u>centripetal</u>

centripetal is a direction

$\left.\begin{array}{l} x \\ y \end{array}\right\}$ can be directions

$\left[\begin{array}{l} \text{centripetal} \\ \text{tangential} \end{array}\right.$

$$\vec{a} = -\omega^2 \vec{r}$$

acceleration for uniform circular motion is always toward the center of the circle.

$\Rightarrow$ We call it

centripetal acceleration

$\vec{a} = -\omega^2 \vec{r}$

Book usully writes

$$\boxed{a_c = \frac{v^2}{r}}$$

$$|\vec{a}| = |-\omega^2 \vec{r}|$$

$$a = +r\omega^2$$

$$= r\left(\frac{v}{r}\right)^2$$

$$= \frac{v^2}{r}$$

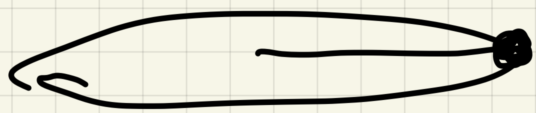
We can also say

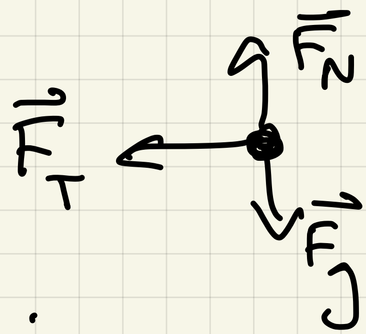
$$\sum F_c = m a_c$$

Σ of fores in centripetal dir

acc in centripetal dir.

Ex: A 2 kg mass on the end of a string spins in a circle of radius 40 cm.





The tension in the string is 45 N.

If it's moving in uniform circular motion, what is the object's speed?

$$\Sigma F_c = m a_c$$

$$F_T = m\frac{v^2}{r}$$

$$v = \sqrt{\frac{r F_T}{m}} = \sqrt{\frac{(.4\,m)\,45\,N}{2\,kg}}$$

$$\boxed{v = 3\ m/s}$$