IC18: 11/16 In Class –Simple Harmonic Motion

Summary of SHM so far:

Simple Harmonic Motion (SHM) is defined as motion where the force or acceleration is proportional to the negative of the displacement.

$$F_x \propto -x \qquad a_x \propto -x$$

The constant that makes the proportionality an equality is $m\omega^2$ for force, or ω^2 for acceleration.

$$F_x = -m\omega^2 x \qquad a_x = -\omega^2 x$$

where ω is the angular velocity which is the same as the angular frequency.

Recall that for a spring, $\omega = \sqrt{\frac{k}{m}}$, and (today I showed) for a simple pendulum, $\omega = \sqrt{\frac{g}{\ell}}$

1. A simple pendulum is made with a mass hanging from a string of length 1.5 m. If the mass is displaced from equilibrium, what will be the period of its motion?

2. We saw in class that a simple pendulum is only SHM when $\sin\theta \approx \theta$. Check a few in radians, fill in this table:

θ in deg	θ in rad	$\sin\theta$
30		
10		
1		
0.1		

   I pretty much use about 30° as my cutoff for small angles. Unless I know I'm going to be more precise than 4% uncertainty.

3. A mass of 1.5 kg hangs at rest from the end of a spring of constant 200N/m. I displace it 10cm and let it go.

   (a) What is the period of its motion?

   (b) Write the equation of motion for the mass. (x (or y) as a function of t.)

(c) What happens to the motion (what is different, what is the same) if I pull it back twice as far and let go?

(d) Write the equation for the second motion as well.

4. We have not yet found what the total energy of a SHO is. To prove conservation of energy, we need to show that energy is constant. It turns out the constant is $E = \frac{1}{2}m\omega^2A^2$, which is the same as: $E = \frac{1}{2}kA^2$ for a spring. Let's do the spring. You can always go from k to $m\omega^2$ to make it general.

   (a) Begin with $x = A\cos\omega t$. When you take the derivative, you get $v = -\omega A\sin\omega t$. Use this x and v in your expression for total energy: $E = KE + PE$. (Use the same expressions for KE and PE you'd have used in chapter 6.)

   (b) Do some algebra and show that $E = \frac{1}{2}kA^2$

   (c) Try to explain in words why this answer makes sense.

5. In symbols (see the previous problem), what is the maximum speed of a SHO? Express the total energy, E, in terms of v_{max}. Explain in words why this one makes sense.