Velocity and Speed

by Jessica Kintner

velocity, $\vec{v}$ ≡ the derivative of position wrt time
≡ time rate of change in position

$$\boxed{\vec{v} \equiv \frac{d\vec{x}}{dt}}$$

- it is a vector
- units m/s

(I have a couple videos on calculus review.)

Two methods to find derivative
1.) Graphical
2.) Analytical

Graphical method:

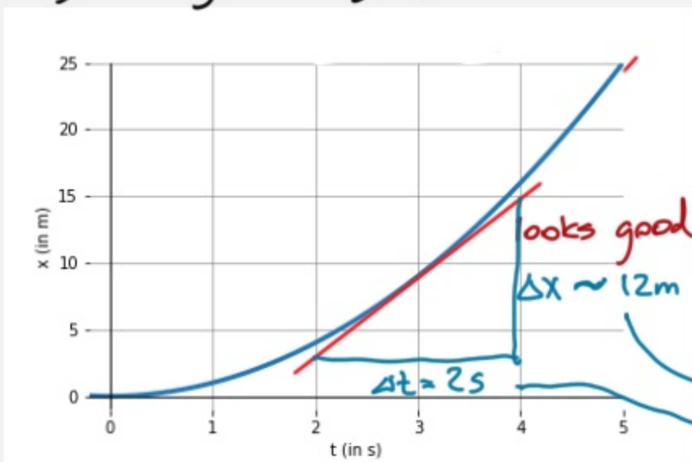
The velocity is the slope of the tangent to the curve at the point of interest.

Analytical Method

- use shortcuts you learned in Calc I.

f	$\frac{df}{dt}$
A	0
At^n	nAt^{n-1}
$A\cos\omega t$	$-\omega A\sin\omega t$
$B\sin\omega t$	$\omega B\cos\omega t$

Example 1: An object's motion is given by this graph:



a) What is the object's velocity at $t = 3s$?

Answer: Draw the tangent to the curve at $t = 3s$ - then find its slope.

How to draw a tangent line

The tangent line should just touch the curve at the point of interest. It should not cross the curve at or near the point.

$$\text{slope of tangent} = \frac{\Delta \vec{x}}{\Delta t} = \frac{12m}{2s} = \boxed{6\ m/s}$$

b.) The equation of motion for this graph is

$$x = 3t^2$$

Check your answer to a) analytically.

Answer: for $x = t^2$

$$v = \frac{dx}{dt} = 2t^{(2-1)} = 2t$$

at $t = 3s$

$$= \boxed{6\ m/s}$$

A word on units in equations

Rarely do it!

$x = t^2$ means { where x is in m and t is in s is understood!

$$x = t^2 \frac{m}{s^2}$$

The danger is that m looks like the symbol for mass.

If things are not in SI (metric) units, that's when it's explicit.

Example 2: A ball has a position given by:

$$x = 10 + 12t - 5t^2$$

a) What is the ball's velocity at any time, t?

b) At $t = 1s$?

c.) At $t = 2s$?

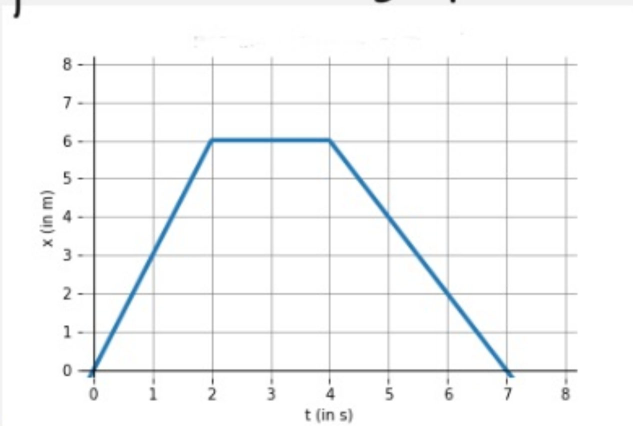
Answers:

a.) $v \equiv \frac{dx}{dt} = 12 - 2(5t) = \boxed{12 - 10t}$

b.) $v(1s) = 12 - 10 = \boxed{2\ m/s}$

c.) $v(2s) = 12 - 10(2s) = 12 - 20 = \boxed{-8\ \frac{m}{s}}$

Example 3: An object has a position vs time graph as shown

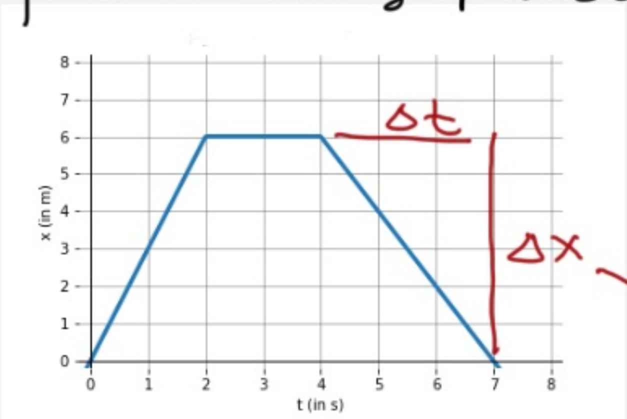


a) What is the object's velocity at $t = 1s$?

b) At $t = 3s$?

c) At $t = 5s$?

Example 3: An object has a position vs time graph as shown



a) What is the object's velocity at $t = 1s$?

b) At $t = 3s$?

c) At $t = 5s$?

Answers

a.) at $t = 1s$, v = slope tangent at $t = 1s$

$$\vec{v} = \frac{\Delta \vec{x}}{\Delta t} = \frac{6-0\ m}{2-0\ s} = \boxed{3\,m/s}$$

b.) At $t = 3s$, tangent would be flat (horizontal) line

$\boxed{\text{slope} = 0} = \vec{v} = 0\ \frac{m}{s}$

c.) At $t = 5s$, the tangent is same as line

$$\vec{v} = \frac{\Delta \vec{x}}{\Delta t} = \frac{\vec{x}_f - \vec{x}_i}{t_f - t_i} = \frac{0-6m}{7-4s} = -\frac{6m}{3s} = \boxed{-2\,m/s}$$

speed $\equiv$ magnitude of the velocity.

speed $\equiv |\vec{v}| = v$

- a scalar, always +
  (speed limit, direction does not matter)
- units m/s

In that last example, when $\vec{v} = -2\,m/s$, the speed is $+2\,m/s$

Note: The velocity and speed defined in this video refer to what is also called instantaneous velocity and instantaneous speed.

(As opposed to average velocity and average speed --> coming in a later video.)